Profit Analysis of a Two-Unit Cold Standby Centrifuge System with Single Repairman

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Abstract— This paper deals with two unit centrifuge system where faults are characterized as major and minor fault. It is assumed that system leads to partial failure state on occurrence of a minor fault whereas on occurrence of a major fault it leads to complete failure. On occurrence of a failure in the system, either the repairman carry out the repair of the components involved or the unit wait for repair if the repairman is busy. Various measures of system effectiveness are obtained by using Markov processes and regenerative point technique. The analysis of the system is carried out on the basis of the graphical studies and conclusions are drawn regarding the reliability of the system.

Index Terms— Centrifuge System, MTSF, Expected Uptime, Profit, Markov Process, Regenerative Point Technique

1 INTRODUCTION

IN the present scenario filtration and purification plays a very important role in the modern society pertaining to the health of the human being and the qualities of the products used by them. A large number of equipments or systems of equipments are involved in the industries to meet out the requirements of such products. One such system is a centrifuge system used for separation of two objects having different type of density. Centrifuge system is being used in Refineries for oil purification, in milk plants to extract the fats, in laboratories for blood fractionation and wine clarification etc. A centrifuge system involves equipment, generally driven by an electric motor that puts an object in rotation around a fixed axis, applying a force perpendicular to the axis. Thus the reliability and cost of the centrifuge system plays a very significant role in the situations wherever these are used and hence need to be analyzed.

The working of a centrifuge system in Jindal Drilling and Industries Ltd., BKC Bandra East Mumbai was observed and the real data on failure/faults, inspection, maintenance and repairs, etc. for the system was collected. It was found that this system can have various kinds of minor and major faults including as motor burnt, gear damage, bearing damage, alignment etc. that leads to failure/degradation of the system.

Many researchers in the field of reliability modeling including Gupta and Kumar (1983), Gopalan and Murlidhar (1991), Tuteja et al (2001), Taneja et al (2004), Taneja and Parashar (2007), Gupta et al (2008), Kumar et al (2010), etc. analyzed a large number of one unit/ two unit systems. Kumar and Bhatia (2011, 2012, 2013) discussed the behaviour of the single unit centrifuge system considering the concepts of inspections, halt of system, degradation, minor/major faults, neglected faults, online/offline maintenances, repairs of the faults etc.

Various values of failure rates, repair rates etc. estimated from the data collected for centrifuge system are:

Estimated value of failure rate due to occurrence of major faults (λ_1) = 0.0023

Estimated value of failure rate due to occurrence of minor faults (λ_2) = 0.0056

Estimated repair rate on occurrence of minor faults (β_1) = 0.190

Estimated repair rate on occurrence of repairable major faults (β_2) = 0.328

In reliability modeling, none of the researchers have analyzed such two unit centrifuge system considering various faults. To fill up this gap, the present paper analyses a two unit centrifuge system considering minor and major faults. Whereas faults such as leakage of seal, motor overheating, liquid seal broken, alignment etc. are considered as minor faults and faults such as motor burnt, gear damage, bearing faults, O-ring damage etc. are considered as major faults. It is assumed that minor fault leads to down state while major fault leads to complete failure of the system. On a failure of the system, the single repairman reaches to the system in negligible time and carries out the repair of the components involved. Various measures of system effectiveness such as mean sojourn time, MTSF, expected up time, expected down time of the system and busy period of the repairman are obtained using Markov processes and regenerative point technique. The conclusions

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regarding reliability and profit of the system are given on the basis of graphical studies.

2 OTHER ASSUMPTIONS

a) Faults are self- announcing.
b) There is a single repairman facility.
c) After each repair the system is as good as new.
d) On major fault repairman is coming instantly.
e) During online repair / waiting for repair there may be occurrence of major fault.
f) The failure time distributions are exponential while other time distributions are general.
g) Switching is perfectly done on occurrence of major fault.
h) All the random variables are mutually independent.

3 NOTATIONS

λ_1 / λ_2	Rate of occurrence of major / minor failure
$g_1(t)$ / $G_1(t)$	p.d.f. / c.d.f. of times to repair the unit at failed state
$g_2(t)$ / $G_2(t)$	p.d.f. / c.d.f. of times to repair the unit at failed state
O_r / O_w / O_{cs}	Operative unit under repair / waiting / cold standby
F_r / F_w	Failed unit under repair / waiting

4 THE MODEL

A state-transition diagram in fig. 1 shows various states of transition of the system. The epochs of entry into states 0, 1 and 2 are regeneration points and thus these are regenerative states. The states 3 and 4 are failed state.

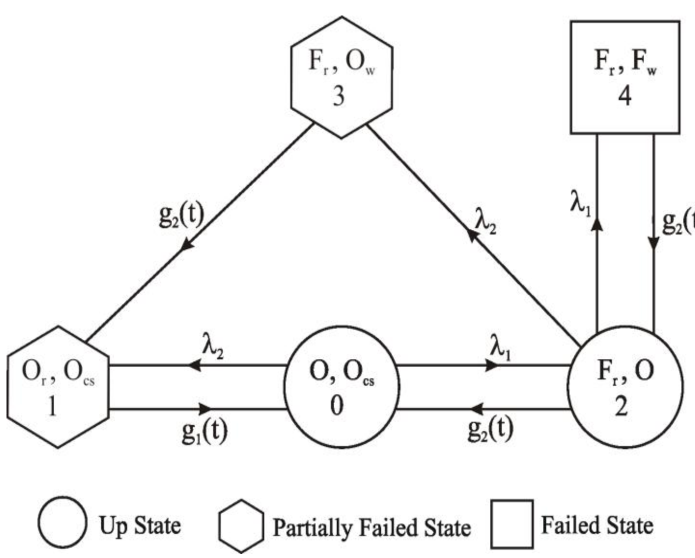

Fig. 1 State Transition Diagram

5 TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

The transition probabilities are

$$dQ_{01}(t) = \lambda_2 e^{-(\lambda_1+\lambda_2)t}dt \qquad dQ_{02}(t) = \lambda_1 e^{-(\lambda_1+\lambda_2)t}dt$$
$$dQ_{10}(t) = g_1(t)dt \qquad dQ_{20}(t) = e^{-(\lambda_1+\lambda_2)t} g_2(t)dt$$
$$dQ_{23}(t) = \lambda_2 e^{-(\lambda_1+\lambda_2)t}\overline{G_2}(t)dt \qquad dQ_{24}(t) = \lambda_1 e^{-(\lambda_1+\lambda_2)t}\overline{G_2}(t)dt$$
$$dQ_{22}^4(t) = \left(\lambda_1 e^{-(\lambda_1+\lambda_2)t} \copyright 1\right) g_2(t)dt \qquad dQ_{31}(t) = g_2(t)dt$$

The non-zero elements p_{ij} are $p_{ij} = \lim_{s\to 0} Q_{ij}^{**}(s)$

$$p_{01} = \frac{\lambda_2}{\lambda_1+\lambda_2} \qquad p_{02} = \frac{\lambda_1}{\lambda_1+\lambda_2}$$
$$p_{10} = g_1^*(0) \qquad p_{20} = g_2^*(\lambda_1+\lambda_2)$$
$$p_{23} = \frac{\lambda_2\left[1-g_2^*(\lambda_1+\lambda_2)\right]}{\lambda_1+\lambda_2} \qquad p_{31} = g_2^*(0)$$
$$p_{22}^4 = \frac{\lambda_1\left[1-g_2^*(\lambda_1+\lambda_2)\right]}{\lambda_1+\lambda_2} = p_{24}$$

By these transition probabilities, it can be verified that

$$p_{01}+p_{02}=1, \qquad p_{20}+p_{23}+p_{24}=1,$$
$$p_{20}+p_{23}+p_{22}^4=1, \qquad p_{10}=p_{31}=p_{42}=1$$

The mean sojourn time in the regenerative state i(μ_i) is defined as the time of stay in that state before transition to any other state then we have

$$\mu_0 = \frac{1}{\lambda_1+\lambda_2} \qquad \mu_1 = g_1^{*\prime}(0)$$
$$\mu_2 = \frac{1-g_2^*(\lambda_1+\lambda_2)}{(\lambda_1+\lambda_2)} \qquad \mu_3 = g_2^{*\prime}(0)$$

The unconditional mean time taken by the system to transit for any regenerative state j, when it is counted from epoch of entrance into that state i, is mathematically stated as-

$$m_{ij} = \int_0^\infty t\,dQ_{ij}(t) = -q_{ij}^{*\prime}(0)$$, Thus-

$$m_{01}+m_{02}=\mu_0 \qquad m_{10}=\mu_1 \qquad m_{31}=\mu_3$$
$$m_{20}+m_{23}+m_{24}=\mu_2 \qquad m_{20}+m_{23}+m_{22}^4=k_1$$

where

$$k_1 = -g_2^{*\prime}(0)$$

6 OTHER MEASURES OF SYSTEM EFFECTIVENESS

Using probabilistic arguments for regenerative processes, various recursive relations are obtained and are solved to derive important measures of the system effectiveness that are as given below:

Mean time to system failure $(T_0) = N/D$

Expected up time of the system with full capacity

$(AF_0) = N_1/ D_1$

Expected up time of the system with reduced capacity

$(AR_0) = N_2/ D_1$

Busy period of repair man (Repair time only)

$(B_r) = N_3/ D_1$

Where

$$N = \mu_0 + \mu_1 (p_{01} + p_{02}p_{23}) + p_{02}\mu_2 + p_{20}p_{23}\mu_3$$

$$D = 1 - p_{01}p_{10} - p_{02}(p_{20} + p_{23}p_{31}p_{10})$$

$$N_1 = \mu_0 (1 - p_{22}^4) + p_{02}\mu_2$$

$$N_2 = p_{01}(1 - p_{22}^4)\mu_1 + p_{02}p_{23}p_{31}\mu_1 + p_{02}p_{23}\mu_3$$

$$N_3 = p_{01}(1 - p_{22}^4)\mu_1 + p_{02}p_{23}p_{31}\mu_1 + p_{02}\mu_2 + p_{02}p_{23}\mu_3$$

$$D_1 = \mu_0 + p_{01}\mu_1 + p_{02}k_1 + p_{02}p_{23}(\mu_1 + \mu_3) - p_{22}^4 (1 + p_{01}\mu_1)$$

7 PROFIT ANALYSIS

The expected profit incurred of the system is

$P = C_0AF_0 + C_1AR_0 – C_2Br – C_3$

where

C_0 = Revenue per unit uptime of the system with full capacity.

C_1 = Revenue per unit uptime of the system with reduced capacity.

C_2 = Cost per unit repair of the failed unit

C_3 = Cost of installation

8 GRAPHICAL ANALYSES

For graphical analysis the following particular cases are considered:

$g_1(t) = \beta_1 e^{-\beta_1 (t)}$ $g_2(t) = \beta_2 e^{-\beta_2 (t)}$

Various graphs are plotted for MTSF, Expected up time and Expected down time and Profit of the system by taking different values of failure rates (λ_1 & λ_2), and repair rates (β_1& β_2).

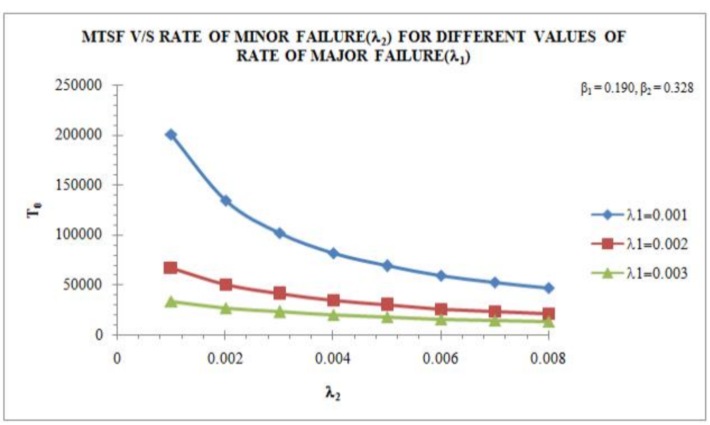

Fig. 2

Fig.2 gives the graph between MTSF (T_0) and the failure rate (λ_2) due to minor faults for different values of failure rate (λ_1) due to major faults. The graph reveals that the MTSF decreases with increase in the values of the failure rates.

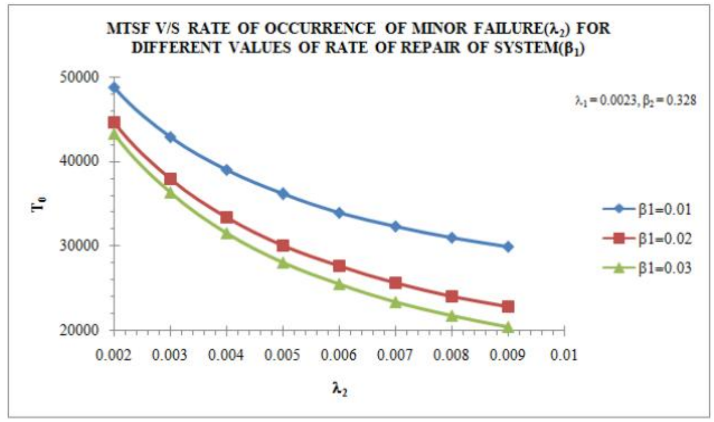

Fig.3

Fig.3 gives the graph between MTSF (T_0) and rate of occurrence of minor failure (λ_2) for different values of rate of repair of the system (β_1). The graph reveals that the MTSF decreases with increase in the values of the failure rate as well as repair rate.

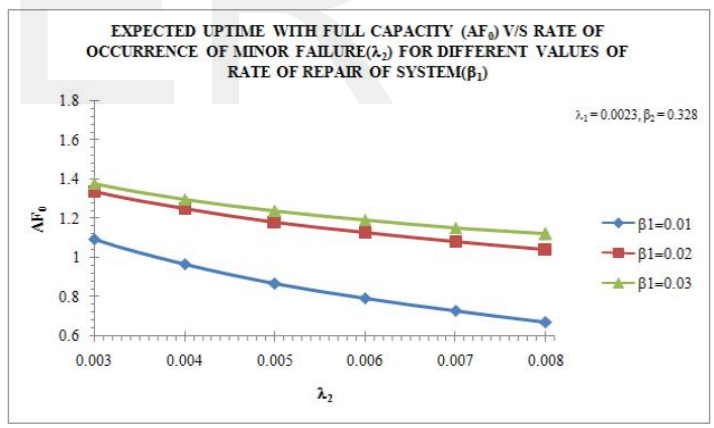

Fig.4

Fig.4 gives the graph between Expected uptime with full capacity (AF_0) and the failure rate of occurrence of minor faults (λ_2) for different values of rate of repair of the system (β_1). The graph reveals that the Expected uptime with full capacity decreases with increase in the values of the failure rate whereas it increases with increase in the values of the rate of repair of the system.

The curves in the Fig.5 show the behavior of the profit with respect to the revenue per unit up time with full capacity (C_0) of the system for the different values of rate of occurrence of major faults (λ_1). It is evident from the graph that profit increases with the increase in revenue per unit up time of the system with full capacity for fixed value of the rate of occurrence of major faults. From the Fig. it may also be observed that for $\lambda_1 = 0.001$, the profit is negative or zero or positive according as C_0 is < or = or > Rs. 7805.345. Hence the system is profitable to the plant whenever $C_0 \geq$ Rs. 7805.345. Similarly, for $\lambda_1 = 0.008$ and $\lambda_1 = 0.015$ respectively the profit is negative or zero or positive according as C_0 is < or = or > Rs. 7982.331 and Rs. 8156.099 respectively. Thus, in these cases, the system is profitable to the plant whenever $C_0 \geq$ Rs. 7982.331 and Rs. 8156.099 respectively.

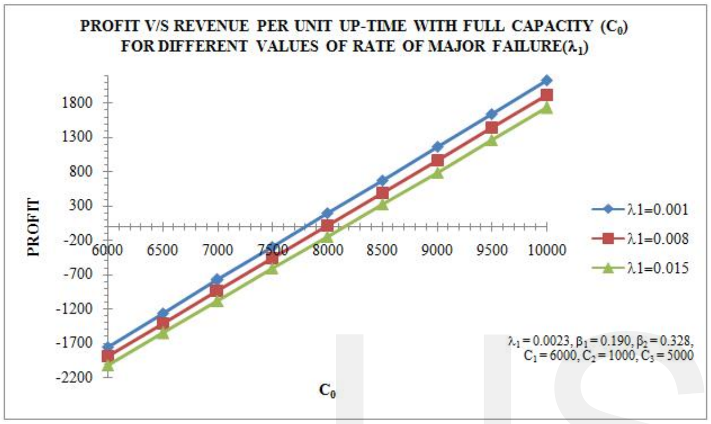

Fig.5

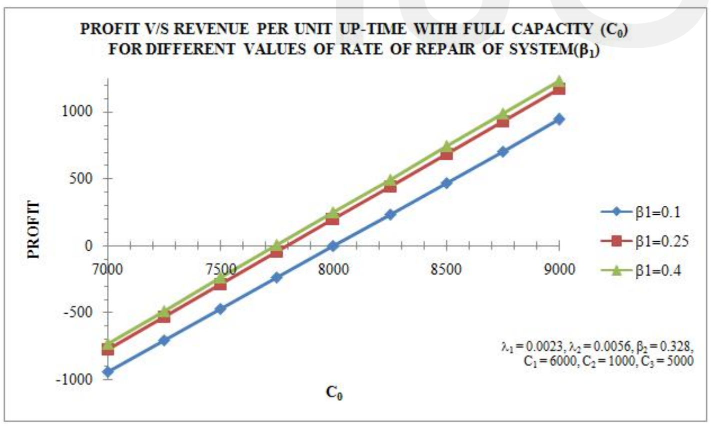

Fig.6

The curves in the Fig.6 show the behavior of the profit with respect to the revenue per unit up time with full capacity (C_0) of the system for the different values of rate of repair of system (β_1). It is evident from the graph that profit increases with the increase in revenue per unit up time of the system with full capacity for fixed value of the rate of repair of system. From the Fig. it may also be observed that for $\beta_1 = 0.1$, the profit is negative or zero or positive according as C_0 is < or = or > Rs. 8000.276. Hence the system is profitable to the plant whenever $C_0 \geq$ Rs. 8000.276. Similarly, for $\beta_1 = 0.25$ and $\beta_1 = 0.4$ respectively the profit is negative or zero or positive according as C_0 is < or = or > Rs. 7795.315 and Rs. 7744.076 respectively. Thus, in these cases, the system is profitable to the plant whenever $C_0 \geq$ Rs. 7795.315 and Rs. 7744.076 respectively.

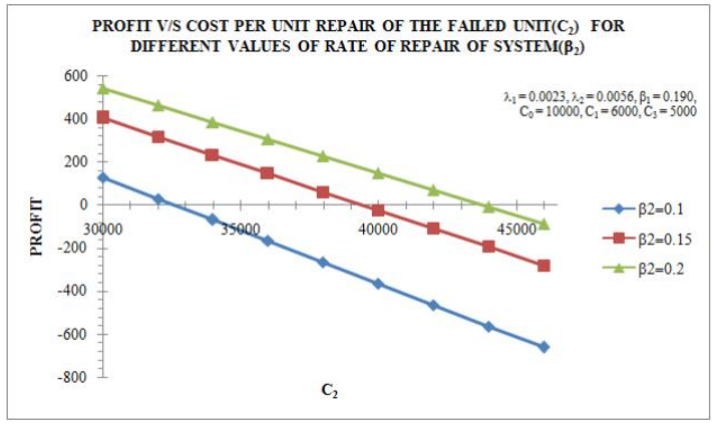

Fig.7

The curves in the Fig.7 show the behavior of the profit with respect to the Cost per unit repair of the failed unit (C_2) of the system for the different values of rate of repair of system (β_2). It is evident from the graph that profit increases with the increase in revenue per unit up time of the system with full capacity for fixed value of the rate of repair of system. From the Fig.7 it may also be observed that for $\beta_1 = 0.1$, the profit is negative or zero or positive according as C_0 is < or = or > Rs. 32600.37. Hence the system is profitable to the plant whenever $C_0 \geq$ Rs. 32600.37. Similarly, for $\beta_1 = 0.25$ and $\beta_1 = 0.4$ respectively the profit is negative or zero or positive according as C_0 is < or = or > Rs. 39435.60 and Rs. 43811.98 respectively. Thus, in these cases, the system is profitable to the plant whenever $C_0 \geq$ Rs. 39435.6 and Rs. 43811.98 respectively.

9 CONCLUSIONS

The analysis discussed above shows that the mean time to system failure and the expected uptime with full capacity of the centrifuge system decreases with the increase in the values of the rate of occurrence of minor/major faults. For the profit of the system, the analysis stated various cut-off points of the revenue per unit up time and cost per unit repair of the failed unit to enhance the profit of the system.

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